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Estimation of Shear Modulus for Layered Composite Panels by Finite Element Method

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This paper discusses the numerical analysis of panel shear rigidity using computer simulation with finite element method (FEM). The analytical object was a layered composite panel consisting of 5-ply yellow meranti (*Shorea spp.*) plywood inserted with two pieces of graphite fiber cross-woven cloths, called graphite fiber-reinforced plywood. Seven patterns of reinforced plywoods with different constitutions, including a conventional plywood without reinforcement, were analyzed.

A four-node bilinear thick shell element capable of modeling the behavior of layered composite panel was used. For simplicity, the square graphite fiber-reinforced plywood was considered to be subjected to biaxial stress with equal but opposite values so as to create a state of pure shear for a certain part within the plywood.

The calculated results indicate that the orientation of reinforcing cloths had significant effect on the panel shear properties, where 45° orientation of reinforcing cloths is the most effective for improving the rigidity of panel shear. However, there are no differences among the panel shear rigidity of three inserting positions of reinforcing cloths. These results are well in agreement with the published experimental results determined by vibration method. Thus, it could be concluded that this analytical method can be used for predicting the shear modulus of layered composite panel with reasonable reliability.

This numerical analysis was further applied to clarify the relationship between shear modulus of reinforced plywood and the orientation of reinforcing cloth quantitatively. In addition, the stress distribution in each lamina with different orientations of reinforcing cloth was also evaluated. It is hoped that these results could lay a foundation for designing the structural composite panels with high performance in shear.

Key words: fiber-reinforced plywood, structural panel, shear modulus, finite element method, numerical analysis.

1. Introduction

Shear rigidity is one of the most important properties of panel for structural application. It is usually determined based on American Standards for Testing and Materials (ASTM) D 805 or Swedish Larsson-Wästlund (LW) panel testing method. However, the values obtained from these two methods are quite different, due to the differences in equipment proper and the restrained conditions[1], amongst others. Recently, non-destructive testing methods such as vibration[2] or ultrasonic techniques[3] are widely used in measuring the elastic constants of wood-based composites, but these can only be applied where actual specimens are available.

This study attempts to introduce a simple way to predict the elastic constants of wood composite panel through numerical analysis. This initial attempt focuses on the shearing modulus of layered composite panel. Computer simulation of finite element method was used to determine the shear rigidity of different analytical models. In order to confirm the reliability of the analytical results, a series of published experimental data[4] on the fiber-reinforced plywoods were referred. Seven types of fiber-reinforced plywoods with different constitutions were taken as the analytical objects.

2. Analytical method

Fig. 1 illustrates a two dimensional system, where a square plate is subjected to biaxial state of stress. Assuming the tensile stress σ_x to be equal to the compressive stress σ_y, then the normal stress on the sides of plane abcd, with sides parallel to the x-axis and inclined at 45° to the y-axis, is canceled, and the shearing stress (τ) is

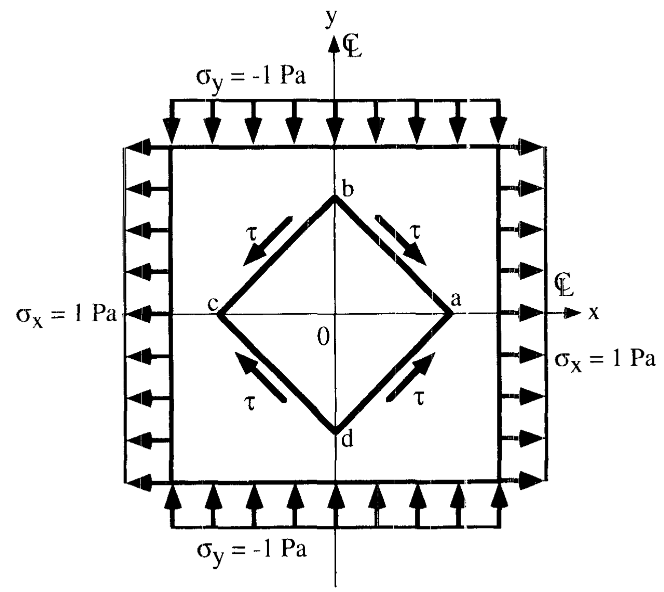

Fig. 1. Condition of pure shear.

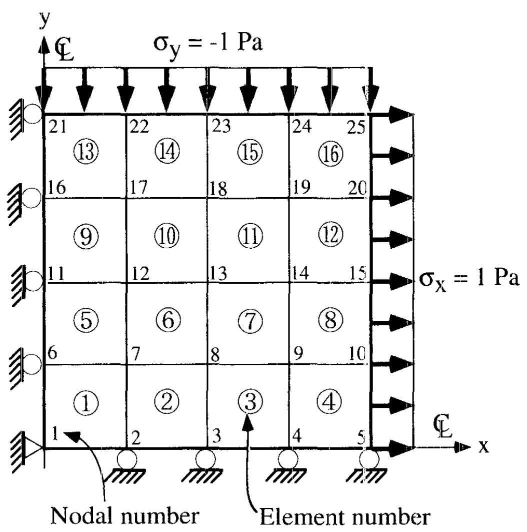

Fig. 2. Analytical model with element division.

$$\tau=\sigma_x=-\sigma_y \qquad (1)$$

This plane represents a condition of pure shear. Subsequently, the shear strain (γ) and shear modulus (G) could be calculated from the strain along the x-axis (ε_x) and y-axis (ε_y) in the plane using the following formulas:

$$\gamma=|\varepsilon_x-\varepsilon_y| \qquad (2)$$

$$G=\tau/\gamma \qquad (3)$$

The layered composite panel was analyzed based on this model. Since the configuration is symmetrical, the analysis was done on a quarter of the plate, which is divided into 16 elements with a total of 25 nodes, as shown in Fig. 2.

The element used is a 4-node bilinear thick shell element with three translational and rotational degrees-of-freedom at each node. The stresses applied are: $\sigma_x=1$ Pa and $\sigma_y=-1$ Pa. In order to fully restrain the rigid body modes without introducing any elastic constraints, a special set of boundary condition is applied. Three translational and two rotational out-of plane degrees-of-freedom are suppressed at node No.1, and the translational degrees-of-freedom in x-axis is suppressed along the entire left edge. Since the lay-up is symmetrical, only in-plane deformations are expected. The specification of additional rotational constraints at the left edge is irrelevant.

The analytical object is a layered composite panel which consists of 5-ply phenol resorcinol formaldehyde bonded plywood inserted with two pieces of phenol-formaldehyde resin impregnated graphite fiber cross-woven cloths, termed as graphite fiber-reinforced plywood hereafter. The plywood was composed of 5 layers of yellow meranti (*Shorea spp.*) rotary veneers (3.05 mm thick)

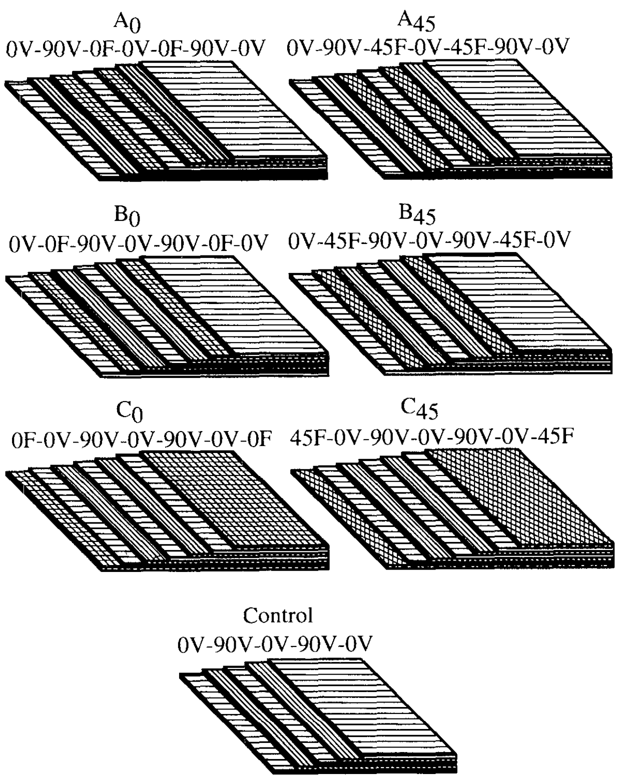

Fig. 3. Seven types of graphite fiber reinforced plywoods.
Notes: V: Veneer, F: Graphite fiber cloth, the numbers before V or F are the orientations of grain or fiber to face grain.

Table 1. The fundamental properties of the basic materials used in FEM calculation.

Material	Young's modulus (GPa)		Shear modulus (GPa)	Poisson's ratio
Veneer[5] Yellow meranti (*Shorea spp.*)	E_L 17.0	E_T 1.24	G_{LT} 0.51	μ_{LT} 0.61
Graphite fiber cloth (TR3110M, Mitsubishi Rayon)	E_H 73.0	E_V 70.0	G_{HV} 12.2	μ_{HV} 0.30

Notes: The subscript letters L and T denote the longitudinal and tangential directions of wood, respectively. While H and V indicate the warp and weft directions of graphite fiber cloth, respectively. The mechanical properties of graphite fiber cloth were given by Mitsubishi Rayon Ltd..

and two pieces of graphite fiber cross-woven cloths (0.19 mm). In order to investigate the analytical accuracy, three sets each of graphite fiber-reinforced plywood with two different reinforcing cloth orientations were taken as the analytical objects.

Seven types of reinforced plywoods with different constitutions (types Control, A_0, B_0, C_0, A_{45}, B_{45} and C_{45}) are shown in Fig. 3. The subscript numbers of 0 and 45 indicate the two groups of fiber reinforced plywood where the angle between the graphite fiber direction and the grain of veneer is 0° and 45°, respectively. The capital letters A, B and C represent three lay-ups of veneers and graphite fiber cross-woven cloths, where type A: two pieces of fiber cloths in the 3rd and 5th plies, type B:two pieces of fiber cloths in 2nd and 6th plies, and type C:two pieces fo fiber cloths on each surface. Type Control is the conventional plywood without graphite fiber reinforcement. Assuming the graphite fiber-reinforced plywood to be perfectly elastic and homogeneous, both of these components can be considered as orthotropic materials. However, the mechanical properties of veneer in the state of plywood are quite different from those of the solid wood and free veneer, because of the formation of knife checks during veneer peeling, addition of resin adhesive, hot-pressing and other manufacturing processes. The elastic constants of veneer and fiber cloth used in the computation, as based on the literature[5], and tabulated in Table 1. All of the properties are in their principal structural directions. According to the Hooke's Law[6] for orthotropic materials, the elastic constants of graphite fiber cross-woven cloth along optional orientations could be calculated through transformation from the principal direction.

3. Results and Discussion

The comparison between the referred experimental[4] and calculated panel shear modulus (G) of reinforced plywood is shown in Fig. 4. The experimental values were determined using vibration technique. All of the analytical values, except that without reinforcement, are slightly higher than those obtained from the experiment. The differences between these values were within 10%. This is because the theoretical analysis was done based on the assumption that the mechanical properties of the laminae used were ideal and uniform, which in fact might be violated in certain situations. Even so, it is evident from Fig. 4 that both the analytical and referred experimental results agree well with each other. It could thus be put concretely as follows :

a. Insertion of graphite fiber cross-woven cloths is obviously effctive for plywood reinforce-

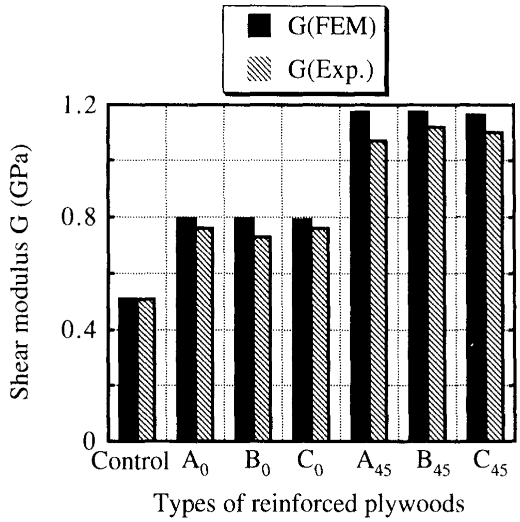

Fig. 4. Comparison of shear modulus between calculated and experimental results.
Notes: G(FEM): Shear moduli obtained by FEM, G(Exp.): Shear moduli determined using vibration technique[4].

ment. The shear rigidity (G) of fiber reinforced plywoods with 0° orientation reinforcement (types A_0, B_0 and C_0) were 1.5 times of those without reinforcement (Control), while those with 45° orientation reinforcement (types A_{45}, B_{45} and C_{45}) recorded shear moduli of 2.3 times that of Control.

b. The position of fiber cloth insertion in the plywood had no significant effect on the shear modulus of reinforced plywood. However, the reinforcing direction is of significant importance. The shear moduli of types A_{45}, B_{45} and C_{45} fiber reinforced plywood were 1.4 times of those of types A_0, B_0 and C_0.

The following simple formula[7] was proposed for estimating the effective shear modulus of layered composite based on the properties of its laminae:

$$G=\frac{1}{h}\sum_{i=1}^{n}G_i h_i \tag{4}$$

where G_i and h_i are the shear modulus and the thickness of i layer lamina, respectively, and h is the total thickness of the layered composite panel. For the reinforced plywood in this study,

$$G=\frac{1}{h}(5G_{veneer}h_{veneer}+2G_{veneer}h_{veneer}) \tag{5}$$

where the shear modulus of veneer $G_{veneer}=0.51$ GPa, veneer thickness $h_{veneer}=3.05$ mm, the thickness of reinforcing cloth $h_{cloth}=0.19$ mm, total thickness $h=15.64$ mm, and the shear modulus of reinforcing cloth G_{cloth} is a variable depending on the reinforcing orientation (θ) as follows[6]:

$$G_{cloth}=\frac{1}{\left(\frac{1+2\mu_{HV}}{E_H}+\frac{1}{E_V}\right)\sin 2\theta+\frac{\cos^2 2\theta}{G_{HV}}}$$
$$=\frac{1}{0.036\sin^2\theta+0.082\cos^2 2\theta}$$

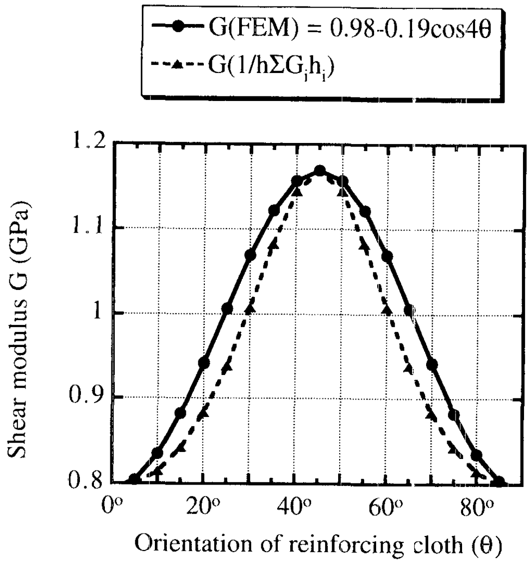

Fig. 5. Relationship between shear modulus and orientation of reinforcing cloth.

$$=\frac{1}{0.059+0.023\cos 4\theta} \tag{6}$$

where the subscript letters H and V indicate the warp and weft directions of graphite fiber cloth, respectively. The mechanical properties of graphite fiber cloth were given by Mitsubishi Rayon Ltd..

By substituting formula (6) into formula (5), the corresponding shear modulus of reinforced plywood G could be determined by the following formula:

$$G=0.5+\frac{0.024}{0.059+0.023\cos 4\theta}$$

$$=0.5+\frac{1}{2.458+0.958\cos 4\theta} \tag{7}$$

The shear moduli obtained from FEM and formula (7) were plotted in Fig. 5 for comparison. Despite the small difference between these two groups, they exhibit acommon trend of relationships between shear modulus (G) and oriented angle of inserted graphite fiber cross-woven cloths (θ). The relationship between G and θ from the results calculated from FEM is shown as follows:

$$G(FEM)=0.98-0.19\cos 4\theta \tag{8}$$

The coefficient of correlation reached 1.0, and the differences of G between formulas (7) and (8) were within 8%.

Therefore, it is clear that the shear moduli of graphite fiber reinforced plywood rely upon the reinforcing orientation greatly. The peak of shear modulus occurred at 45° reinforcing orientation, but decreased with deviation away from 45° orientation. The lowest shear resistance appeared at 0° or 90° reinforcing orientation. Therefore, the effect of reinforcement is greatly enhanced by inserting the reinforcing cloths at an orientation angle of 45°.

It can thus be concluded that the shear modulus of layered composite panels can be estimated with good accuracy and high reliability using finite element method (FEM) computer simulation.

The shearing stress distribution in veneers (τ_V) and fiber cross-woven cloths (τ_F) of the fiber

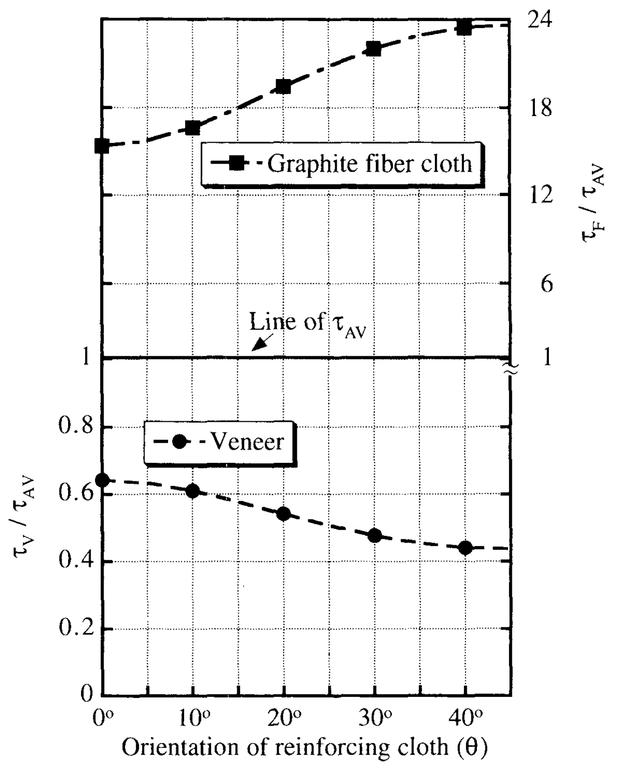

Fig. 6. Comparison of shearing stress among different reinforced plywoods.
Notes: Superscript letters of V, F and AV represent veneer, graphite fiber cloth and the mean value in reinforced plywood, respectively.

reinforced plywoods varied with the orientation angle of inserted graphite fiber cloths (Fig. 6). τ_V is always lower than the average shearing stress in reinforced plywood (τ_{AV}), but the opposite is true for τ_F. τ_V decreased but τ_F increased with increasing orientation angle up to 45° orientation. The ratio of $\tau_F : \tau_V$ increased from 23.9 at 0° orientation up to 54.2 at 45° orientation.

4. Conclusions

The results of this study could be summarized as follows:

Based on the fact that the analytical and experimental results agree very well with each other, it is concluded that computer simulation using finite element method (FEM) is reliable for evaluating the shear modulus of layered composite panels.

The shear rigidity (G) of plywood could be improved by inserting graphite fiber cross-woven cloths into the gluelines. The shear modulus (G) of reinforced plywood is significantly affected by the orientation angle, but not the insertion position of the reinforcing graphite fiber cloth.

The shear moduli (G) of fiber reinforced plywoods with graphite fiber cross-woven cloth inserted at 0° orientation were 1.5 times of those without reinforcement (Control). However, G was further increased by 50% with 45° insertion orientation. Therefore, the reinforcement can be greatly enhanced by inserting the reinforcing layer at 45° orientation.

The numerical analysis was further applied to deduce a simple expression for predicting the shear rigidity of reinforced plywoods with different angles of reinforcement insertion orientaion. The distribution of shearing stress in the veneers (τ_V) and fiber cross-woven cloths (τ_F) of the fiber reinforced plywoods varied with the reinforcement orientatin angle. τ_V is always lower than the average shearing stress in reinforced plywood (τ_{AV}), but the opposite is true for τ_F. τ_V decreased, but τ_F increased with increasing orientation angle of graphite fiber cross-woven cloths inserted. Their ratios increased from 23.9 at 0° orientation up to 54.2 at 45° orientation.

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